## Indian Statistical Institute, Bangalore Centre. Supplementary Exam : Graph Theory

Instructor : Yogeshwaran D.

Date : June 21st, 2016.

Max. points : 50.

## Time Limit : 3 hours.

Give necessary justifications and explanations for all your arguments. If you are citing results from the class, mention it clearly. Results from assignents need to be proved. Answer any five questions. Only the first five answers will be evaluated.

- 1. Prove that the symmetric difference of two different edge-cuts is an edge-cut. (10)
- 2. A greedy algorithm to build an independent set is as follows : Select the vertex with minimum degree and add it to S. Then, delete the vertex and all its neighbours. Now repeat the procedure. Show that S is an independent set of size at least  $\sum_{v \in G} \frac{1}{\deg(v)+1}$ . (10)
- 3. (a) A plane graph is a k-angulation if every face has length k. Let G be a connected graph and a k-angulation on n vertices and m edges. Show that  $m = (n-2)\frac{k}{k-2}$ . (3)
  - (b) Is there a  $K_5$  minor of the Petersen graph ? (2)
  - (c) Show that the Petersen graph has a  $K_{3,3}$  minor. (Hint : Start by deleting one vertex and do not delete any edges.) (5).
- 4. (a) Every tree has at most one perfect matching. Prove or disprove.
  (5)
  - (b) Prove that every maximal matching in a graph has at least  $\alpha'(G)/2$  edges. Here  $\alpha'(G)$  denotes the size of a maximum matching. (5)
- 5. Let M, M' be minimal spanning trees of the graph G with edge-weights w(.). Show that for any  $s \ge 0$ ,

$$|\{e \in M : w(e) = s\}| = |\{e \in M' : w(e) = s\}|.$$
(10)

6. Compute the eigenvalues of the Laplacian and Adjacency matrices of the the cycle graph, the complete graph and the complete bi-partite graph. (10)